Temperature Shift of Neutrino Energy in the External Magnetic Field

M. Gogberashvili, L. Midodashvili, P. Midodashvili

Institute of Physics, Georgian Academy of Sciences, 380077 Tbilisi, Georgia

e-mail: gogber@physics.iberiapac.ge

Abstract

It is evaluated the first order temperature correction to the energy of massive neutrino in the case of weak magnetic field and temperatures $T \ll M_W$.

Keywords: Standard Model, Massive neutrino, Temperature correction, Anomalous magnetic moment.

PACS indeces: 13.10.+q; 13.40.-f

Finite temperature field theories have often been discussed in physics for various reasons [1, 2, 3, 4]. The most fundamental reason is that physics experiments are not carried out at zero temperature; however, since relativistic field theories usually deal with energy scales characterized, as a minimum, by the electron mass, laboratory ambient temperatures qualify as low, and temperature corrections are justifiably neglected. In recent years theoretical interest in finite temperature field theory has grown for other reasons. One is the increased interest in nonabelian gauge theories with spontaneous symmetry breaking; at zero temperature they exhibit a broken-symmetry phase, but at high temperatures the symmetry commonly is restored. This leads into cosmological interests; at the epochs when elementary-particle phenomena were relevant to the evolution of our universe, the temperature was also extremely high, and a satisfactory relation between cosmological evidence and particle physics must take into account temperature effects. A separate interest is related to the increasing availability of data on high energy hadronic collisions; a common feature of models of such processes is the assumption of a temporary state of high-temperature equilibrium before the final state forms. To improve such models a better understanding of finite-temperature field theory will be needed. It is also worth

noticing that recently there has been a great interest in the problem of neutrino interactions with a thermal background [5, 6, 7, 8].

In this paper we present a calculation of finite temperature corrections to massive neutrino energy in the external magnetic field. Our fundamental definition of "finite temperature" is the following: "At finite temperature" means "in the presence of particles". In this case in a thermal-equilibrium distribution exist such particles as photons, electrons, positrons and intermediate bosons. Calculations are carried out in the frame of Weinberg-Salam theory of electroweak interactions [9, 10, 11].

The part of Lagrangian which is connected with dynamic nature of anomalous magnetic moment of Dirac massive neutrino is

$$L = L_W + L_\nu + L_e + L_{int} + L_g, \tag{1}$$

where

$$L_W = -\frac{1}{2}[D_\mu W_\nu^+ - D_\nu W_\mu^+]^2 + \ M^2 W_\mu^+ W^{-\mu} - ie(\partial_\mu A_\nu - \ \partial_\nu A_\mu) W^{+\mu} W^{-\nu} \tag{2}$$

is the Lagrangian of W - boson field interacting with electromagnetic field,

$$L_\nu = \bar{\nu}(i\gamma^\mu \partial_\mu - m_\nu)\nu \tag{3}$$

is the Lagrangian of free neutrino,

$$L_e = \bar{e}(i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu - m)e \tag{4}$$

is the Lagrangian of electron field in the external magnetic field A_μ,

$$L_{int} = \frac{g}{\sqrt{2}}(\bar{\nu_L} W_\mu^+ \gamma^\mu e_L + \bar{e_L} W_\mu^- \gamma^\mu \nu_L) \tag{5}$$

is the Lagrangian of interaction of electron, neutrino and W-boson,

$$L_g = -\frac{1}{\xi}(D^{+\mu} W_\mu^-)(D^{-\alpha} W_\alpha^+) \tag{6}$$

is the part fixing gauge.

In the equations (2) and (6) $D_\mu^\pm = \partial_\mu \pm ieA_\mu$, and right and left components of Dirac bespinors are equal to $\psi_L = \frac{1}{2}(1+\gamma^5)\psi, \quad \psi_R = \frac{1}{2}(1-\gamma^5)\psi$.

It is known that the contribution of charged scalars φ to radiative energy shift of neutrino is smaller than W - boson contribution:

$$\frac{\Delta E_\varphi}{\Delta E_W} \sim \frac{m_e^2}{M_W^2} \ll 1, \tag{7}$$

therefore the corresponding part of the Lagrangian (1) is omitted and we consider only W - boson contribution to the radiative shift of neutrino energy. Let us consider electron neutrino with four-momentum $q = (q_0, \vec{q})$ in the heat "bath" of electrons, positrons, photons and intermediate bosons with zero chemical potential in the external magnetic field

$$A^\mu = (0, 0, xH, 0). \tag{8}$$

The selftime representation of W - boson field propagator in Feynman gauge is given by

$$B_{\mu\alpha}(x, x') = \frac{1}{(4\pi)^2} \int_0^{+\infty} \frac{dt}{t^2}\ B_{\mu\alpha}(t) exp(-iM^2 t) \frac{bt}{\sin(bt)}\ exp[-\frac{i}{4}(x_\parallel^2 - \frac{bx_\perp^2}{\tan(bt)} - 4\Omega)], \tag{9}$$

where

$$\Omega = -b(x_2 - x_2')(x_1 + x_1'), \tag{10}$$

$$x_\perp^2 = (x_1 - x_1')^2 + (x_2 - x_2')^2,$$

$$b = eH, \qquad e > 0,$$

and matrix $B(t)$ has the following nonzero elements:

$$B_{00} = -B_{33} = 1, \qquad B_{22} = B_{11} = -\cos(2y), \tag{11}$$

$$B_{12} = -B_{21} = -\sin(2y), \qquad y = bt.$$

The finite temperature electron propagator in the real-time representation is given by

$$G_\beta(x, x') = -\frac{1}{2\pi} \int_0^{+\infty} d\omega e^{i\omega(t-t')} \sum_s \frac{\psi_n^{(\varepsilon)}(\vec{x}) \bar{\psi}_n^{(\varepsilon)}(\vec{x}')}{\omega + \varepsilon E_n(1 - i\delta)} + i \sum_s e^{-i\varepsilon E_n(t-t')} \frac{\varepsilon \psi_n^{(\varepsilon)}(\vec{x}) \bar{\psi}_n^{(\varepsilon)}(\vec{x}')}{1 + exp(E_n/T)}, \tag{12}$$

where $\psi_n^{(\varepsilon)}(\vec{x})$ is the solution of Dirac equation in the field (8) [12]:

$$\psi_n^{(\varepsilon)}(x) = e^{-i\varepsilon E_n t + ip_2 y + ip_3 z} N \begin{pmatrix} C_1 u_{n-1}(\eta) \\ iC_2 u_n(\eta) \\ C_3 u_{n-1}(\eta) \\ iC_4 u_n(\eta) \end{pmatrix} \tag{13}$$

In this expression $u_n(\eta)$ is Hermite function with argument

$$\eta = \sqrt{eH}(x_1 - \frac{p_2}{\sqrt{eH}}), \tag{14}$$

$C_i \quad (i = 0, 1, 2, 3)$ are spin coefficients [12], and

$$N = (eH)^{1/4}/L \tag{15}$$

is normalization factor with normalization length equal to L.

The shift of neutrino energy is given by

$$\Delta E_\nu(H, T) = -\frac{ig^2}{2} \iint_{-\infty}^{+\infty} d^4x d^4x' \bar{\nu}_L(x) \gamma^\mu G_\beta(x, x') \gamma^\delta \nu_L(x') B_{\mu\delta}(x, x'). \tag{16}$$

Let us consider pure temperature dependence part of expression (16). In the real time formalism [1, 2, 3, 4] one gets automatically the temperature dependence separated from the zero-temperature terms.

If we consider neutrino moving along x - axis, than neutrino wave-function has form:

$$\nu(x) = L^{-3/2} e^{-iE_\nu t + iq_1 x} b_\lambda. \tag{17}$$

In (17) we must take $\lambda = 1$ when the neutrino spin is orientated along the magnetic field, and

$$b_1 = \begin{pmatrix} q_1/\sqrt{2E_\nu(E_\nu - m_\nu)} \\ 0 \\ 0 \\ \sqrt{(E_\nu - m_\nu)/2E_\nu} \end{pmatrix} = \begin{pmatrix} A \\ 0 \\ 0 \\ B \end{pmatrix}. \tag{18}$$

In the case when the neutrino spin is orientated against the magnetic field we must take $\lambda = -1$, and

$$b_{-1} = \begin{pmatrix} 0 \\ q_1/\sqrt{2E_\nu(E_\nu - m_\nu)} \\ \sqrt{(E_\nu - m_\nu)/2E_\nu} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ A \\ B \\ 0 \end{pmatrix}. \tag{19}$$

In the expressions (18) and (19) the neutrino energy is equal to $E_\nu = \sqrt{m_\nu^2 + q_1^2}$. Putting expressions of $G_\beta, B_{\mu\alpha}$ end ν_L in the equation (16) and taking the sum over the spins of intermediate electron states, we get the following formula for temperature shift of neutrino energy:

$$\Delta E_\nu(H, T) = \frac{ig^2}{32\pi^2 L^7} \iint_{-\infty}^{+\infty} d^4x d^4x' \ e^{iE_\nu(x_0 - x_0') - iq_1(x_1 - x_1')} \times \tag{20}$$

$$\sum_{n,\varepsilon,p_2,p_3}\frac{\varepsilon}{1+exp(E_n/T)}e^{i\varepsilon E_n(x_0'-x_0)+ip_2(x_2-x_2')+ip_3(x_3-x_3')}\int_0^{+\infty}\frac{dt}{\cos(eHt)}e^{\frac{i}{2}eH(x_1+x_1')}\times$$
$$\int_{-\infty}^{+\infty}d^4kexp[-ik(x-x')+it(k_0^2-k_3^2-\frac{\tan(eHt)}{eHt}k_\perp^2-M^2)]F(\lambda,n,\varepsilon,p_3,\eta,\eta'),$$

where the sums are taking over the main quantum number n, over the momenta p_2,p_3 and over the sign coefficient $\varepsilon=\pm1$. In (19) the function F has the following form:

$$F=-i\frac{2\varepsilon\sqrt{4\gamma n}}{E_n}AB[u_{n-1}(\eta)u_n(\eta')-u_n(\eta)u_{n-1}(\eta')]-$$
$$-(1+\varepsilon\frac{p_3}{E_n})e^{i2eHt}[A^2(1+\lambda)+B^2(1-\lambda)]u_n(\eta)u_n(\eta')-$$
$$-(1-\varepsilon\frac{p_3}{E_n})e^{-i2eHt}[A^2(1-\lambda)+B^2(1+\lambda)]u_{n-1}(\eta)u_{n-1}(\eta'). \tag{21}$$

Integrating over the variables x_2,x_2',x_3,x_3' and over the x_1' (making substitution $x_1\rightarrow y=\frac{\sqrt{eH}}{2}(x_1-x_1'),\quad x_1'\rightarrow x_1'$), we get expression

$$\Delta E_\nu(H,T)=\frac{ig^2}{32\pi^2\sqrt{eH}}\sum_{n,\varepsilon}\int_{-\infty}^{+\infty}dy\int_o^{+\infty}dt\int_{-\infty}^{+\infty}\frac{d^4k}{\cos(eHt)}[1+exp(E_n/T)]^{-1}\times$$
$$exp\{it[(E_\nu-\varepsilon E_n)^2-k_3^2-\frac{\tan(eHt)}{eHt}k_\perp^2-M^2]+i\frac{2(k_1-q_1)y}{\sqrt{eH}}\}F, \tag{22}$$

where F is the same as in the formula (21), but with the following variables:

$$\eta=y-\frac{k_2}{\sqrt{eH}},\qquad \eta'=-y-\frac{k_2}{\sqrt{eH}},\qquad p_3=-k_3. \tag{23}$$

Using table formula [13]:

$$\int_{-\infty}^{+\infty}e^{-x^2}H_m(x+y)H_n(x+z)dx=2^n\pi^{1/2}m!z^{n-m}L_m^{n-m}(-2yz);\qquad n\geq m, \tag{24}$$

(here $H_n(x)$ is Hermite polynomial and $L_m^{n-m}(x)$ - Laguerre polynomial), integrating over y and making substitution of variebles

$$\frac{k_1-q_1}{\sqrt{eH}}=x_1,\qquad \frac{k_2}{\sqrt{eH}}=x_2,$$
$$\frac{k_3}{\sqrt{eH}}=x_3,\qquad eHt=y. \tag{25}$$

we get

$$\Delta E_\nu(H,T)=\frac{ig^2\sqrt{eH}}{32\pi^2}\sum_{n,\varepsilon}\int_0^{+\infty}\frac{dy}{\cos y}\int_{-\infty}^{+\infty}\frac{(-1)n\varepsilon d^3x}{1+exp(E_n/T)}\times$$
$$exp\{iy[\frac{(E_\nu-\varepsilon E_n)^2}{eH}-x_3^2-\frac{\tan y}{y}((x_1+\frac{q_1}{\sqrt{eH}})^2+x_2^2)-\frac{M^2}{eH}]\}\times$$
$$exp[-iy(x_1^2+x_2^2)][i2x_1R_{n-1,n}L_{n-1}^1(2x_1^2+2x_2^2)+$$
$$+R_{n,n}L_n^0(2x_1^2+2x_2^2)-R_{n-1,n-1}L_{n-1}^0(2x_1^2+2x_2^2)], \tag{26}$$

where

$$R_{n-1,n} = -\frac{i2\varepsilon\sqrt{4\gamma n}AB}{E_n}, \tag{27}$$

$$R_{n-1,n-1} = -(1-\varepsilon\frac{k_3}{E_n})[A^2(1-\lambda)+B^2(1+\lambda)]exp(-i2eHt),$$

$$R_{n,n} = -(1+\varepsilon\frac{k_3}{E_n})[A^2(1+\lambda)+B^2(1-\lambda)]exp(i2eHt),$$

In the plane of variables x_1 and x_2 we can pass to polar coordinates x and φ , where

$$x_1 = x\cos\varphi, \qquad x_2 = x\sin\varphi, \tag{28}$$

$$\int_{-\infty}^{+\infty} dx_1 dx_2 \Rightarrow \int_0^{+\infty} xdx \int_0^{2\pi} xd\varphi.$$

After such substitution we can use well known table formulae [13]:

$$\int_0^{2\pi} d\varphi e^{-ix\sin\varphi+in\varphi} = 2\pi J_n(x); \tag{29}$$

$$\int_0^{\infty} x^{\nu+1}e^{-\beta x^2}L_n^{\nu}(\alpha x^2)J_{\nu}(xy)dx = 2^{-\nu-1}\beta^{-\nu-n-1}(\beta-\alpha)^n y^{\nu}e^{-y^2/4\beta}L_n^{\nu}(\frac{\alpha y^2}{4\beta(\alpha-\beta)}).$$

($J_n(x)$ is Bessel function) and integrate over φ and x .

Finally for the temperature shift of neutrino energy we obtain the following expression

$$\Delta E_{\nu}(H,T) = \Delta E_1(H,T) + \Delta E_2(H,T), \tag{30}$$

where

$$\Delta E_1(H,T) = \frac{ig^2\sqrt{eH}}{16\pi^2}\sum_{n,\varepsilon}\int_0^{+\infty} dy \int_{-\infty}^{+\infty} \varepsilon dx[1+exp(E_n/T)]^{-1} \times \tag{31}$$

$$exp[iy\frac{m_{\nu}^2+m^2+q_1^2-2\varepsilon E_{\nu}E_n-M^2}{eH} - i\sin ye^{-iy}\frac{q_1^2}{eH}] \times$$

$$\{i2\varepsilon\frac{q_1^2}{E_{\nu}E_n}\sin yL_{n-1}^1(z) - \frac{1}{2}[e^{iy}L_n^0(z)+e^{-iy}L_{n-1}^0(z)]\}$$

is part which does not depend on neutrino spin orientation, and

$$\Delta E_2(H,T) = \frac{ig^2\sqrt{eH}}{16\pi^2}\sum_{n,\varepsilon}\int_0^{+\infty} dy \int_{-\infty}^{+\infty} \varepsilon dx[1+exp(E_n/T)]^{-1} \times \tag{32}$$

$$exp[iy\frac{m_{\nu}^2+m^2+q_1^2-2\varepsilon E_{\nu}E_n}{eH} - i\sin ye^{-iy}\frac{q_1^2}{eH}](-\frac{\lambda}{2})\frac{m_{\nu}}{E_{\nu}}[e^{iy}L_n^0(z)-e^{-iy}L_{n-1}^0(z)]$$

is the part which depends on neutrino spin orientation.

Below we present results for various values of temperature and neutrino energies:

1. In the case of weak magnetic fields $H \ll m^2/e$ and low temperatures $T/m \ll 1$, $\Delta E_1(H,T)$ and $\Delta E_2(H,T)$ are of order $exp(-m/T)$; i.e. we have exponential suppression.

2. In the case of weak magnetic fields $H \ll m^2/e$ and temperatures $m \ll T \ll M$ for neutrino energies $E_\nu \ll M^2/T$ we have

$$\Delta E_1(H,T) = \frac{g^2}{4\pi^2}\frac{7\pi^4}{60}E_\nu(\frac{T}{M})^4, \qquad (33)$$
$$\Delta E_2(H,T) = \lambda(\mu_0 H)\frac{4\pi^2}{9}(\frac{T}{M})^2.$$

In this formulae $\mu_0 = 3g^2 m_\nu e/64\pi^2 M^2$ is the statical neutrino magnetic moment.

We would like to thank V. Ch. Zhukovsky, P. A. Eminov, I. M. Ternov and other members of stuff of Moscow University Theoretical Physics Department for usefully discussions and to N. Abuashvili for helping. This work was supported in part by International Science Foundation (ISF) under the Grant No MXL000.

References

[1] D. A. Kirznits, A. D. Linde, Phys. Lett. B42 (1972) 471.

[2] C. W. Bernard, Phys. Rev. D9 (1974) 3312.

[3] L. Dolan, R. Jackiw, Phys. Rev. D9 (1974) 3320.

[4] S. Weinberg, Phys. Rev. D9 (1974) 3357.

[5] D. Notzold, G. Raffelt, Nucl. Phys. B307 (1988) 924;
K. Enqvist, K. Kainulainen, J. Maalampi, Nucl. Phys. B349 (1991) 754.

[6] M. Fukugita, D. Notzold, G. Raffelt, J. Silk, Phys. Rev. Lett. 60 (1988) 879.

[7] V. Ch. Zhukovsky, T. I. Shoniya, P. A. Eminov, Zh. Eksp. Teor. Fiz. 104 (1993) 3269;
V. Ch. Zhukovsky, A. V. Kurilin, P. A. Eminov, Izv. Vyssh. Uchebn. Zaved., Fiz. 30 (1987) 3; Sov. Phys. J. 30 (1987) 1001;

V. Ch. Zhukovsky, A. V. Kurilin, P. G. Midodashvili, P. A. Eminov, Vestn. Mosk. Univ., Fiz. Astron. 29 (1988) 84.

[8] J. C. Olivio, J. F. Nieves, P. B. Pal, Phys. Rev. D40 (1989) 3679;
S. S. Masood. preprint CERN-TH-6622/92.

[9] S. Weinberg, Phys. Rev. Lett. 19 (1976) 1264.

[10] K. Fujikawa, R. E. Shrock, Phys. Rev. Lett. 45 (1980) 961.

[11] A. A. Sokolov, I. M. Ternov, V. Ch. Zhukovski, A. V. Borisov, *Gauge Fields* (Pub.of Moscow University, 1986). In Russian.

[12] A. A. Sokolov, I. M. Ternov, *Relativistic electron* (Nauka, Moscow, 1983). In Russian.

[13] A. P. Prudnikov, V. A. Brichkov, O. I. Marichev, *Integrals and Series* (Nauka, Moscow, 1986). In Russian.